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Modeling the Effect of Political Polarization on the Outcome of Propaganda Battle*

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The paper adds to the literature on propaganda wars. This area attracts practitioners as well as researchers from a variety of fields such as philosophy, social and political science, psychology. It also attracts IT researchers and mathematicians who develop and study models of propaganda wars. In this paper we apply the mathematical model of making choices by individuals to the problem of how the extent of social polarization affects the outcome of the propaganda battle. By the term “propaganda battle” we mean that each member of the society is subject to two competing flows of information. These two flows are generated by two competing parties and each flow consists of propaganda and rumor. That is each party runs propaganda via its own mass-media, and the rumor adds to propaganda as individuals get information from media and transmit it further through interpersonal communications with other individuals. The kind of society is considered which comprises two groups with diametrically opposite fundamental attitudes. The mathematical model has been investigated analytically and numerically. It is shown that moderate political polarization favors the side that runs more intensive propaganda. However, the advantage of stronger propaganda is impaired if the polarization is great enough, because neither media nor individuals can reassure their radical opponents.

Key words: mathematical modeling, propaganda battle, Rashevsky’s neurological scheme, competing rumors.

1. Introduction

The set of issues related to propaganda wars and more broadly, to purposeful spread of information, attracts practitioners as well as researchers from a variety of fields such as philosophy, social and political science, psychology [1,2]. It also attracts IT researchers and mathematicians who develop and study models of propaganda wars.

There is a long history of studying single rumor models [3,4]. In the most general terms, these models assume that we have a closed group of individuals, and at every point in time some of them have a certain piece of information and transmit it to other individuals. Thus, there is a spread of rumor. Today there is an extensive literature on this subject which has been developed mainly as a branch of mathematics, in some separation from social science. A lot of papers develop the approach of [3]. For example, the model with latent, constant recruitment, and varying total population was studied in [5], and the model with several groups of spreaders was considered in [6]. This latter model

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was also used to analyze web search queries. Even more books and papers, for example [7,8], study the spread of single rumors in social networks. A rare example of a single rumor model based on the detailed description of the social mechanism is given in [9]. The models of competing rumors are much less known, although they date back at least to 1977 [10]. One of the few modern papers that considers competing rumors in a social network is [11].

The term rumor, which lies in the center of this strand of literature, implies that some information is being transmitted only from person to person, i.e. through interpersonal communications. The process has some resemblance with the spread of epidemics, so the class of models is called epidemic models of rumor transmission. The more broad term is social contagion.

This approach to spread of information in society is limited by the fact that currently mass-media plays an important (or even leading) role in the spread of information, along with interpersonal communications. In political life, such cases are ubiquitous. If there are two antagonistic sources of information with their own newspapers or TV channels, we can recognize a propaganda war which is far more than just competition of rumors.

The basic model of spread of information through both interpersonal communications and mass media was introduced in [12,13]. The model has the form

$$\frac{dX}{dt} = (\alpha + \beta X)(N - X), \quad X(0) = 0 \qquad (1)$$

Here $X(t)$ is the number of spreaders at time *t*. The parameters α, β are linked to the intensity of mass-media and intensity of propagation of information through interpersonal communications, respectively. This basic model (1) is being developed in several directions. The optimal control problem was introduced and studied in [14,15]; and three factors of spread of information were implemented in the model in [16]. Those factors are incomplete coverage of society by the partisan media, two-step perception of information by individuals, forgetting of information by individuals.

The model of propaganda war [17,18] based on model (1) implies that there are two sources of information. They are antagonistic in the sense that no person can be a spreader of both rumors. The three above-mentioned factors of spread of information were implemented in the model of propaganda war in [19,20].

It should be noted that the theoretical model is not very closely connected with empirical data. The main reason is probably the fact that sociological studies provide real data about the awareness of individuals regarding certain information for only a few moments of time, which is insufficient for proper quantification of the model. However, there is some empirical evidence for the qualitative justification of the models. So, in work [21] devoted to the study of the influence of the media on the genocide in Rwanda in ethnic clashes in 1994, it was shown that the most intense ethnic cleansing happened in the villages who took over the radio station RTLM that promote violence, and in the neighboring ones. Hence, it was concluded that there are two mechanisms for the transmission of information: through radio and interpersonal communication and through the residents of nearby villages.

Finally, we note that models of type (1) (or more complex, but developing the same approaches) are used not only to study the spread of information itself, but also, for example, when modeling the mass distribution of innovative products [22-24].

In this paper, we develop an approach that is focused on a different aspect of information warfare – that is, choosing the positions of the individuals in the confrontation. It is based on the proposed in [25] model built on Rashevsky's neurological scheme [26,27].

This model is applied to the study of the question of how the level of political polarization affects the outcome of the propaganda battle. This question is increasingly relevant firstly due to the development of social media. In recent sociological literature the issue of increasing polarization due to the development of social media and the Internet in general (see, e.g., [28,29]), and the impact of polarization on political events [30] is widely discussed.

The detailed discussion of mathematical formalization of the concept of "a polarized society", is in [31], the main idea in a simple graphic form is presented in [32].

In the present work, polarized society is described using a distribution curve with two high horizontal plateaus. The distance between the gravity centers of these plateaus is taken as a measure of polarization. Thus, the process of increasing polarization has the form of a mutual removal of the plateau from each other.

Further ideas are structured in the following way. Section 2 is devoted to the description of the model, sections 3 and 4 deal respectively with the cases of constant and slowly increasing polarization. Finally, section 5 discusses the sociological meaning of the results.

2. Model

Selection model of the individuals in the propaganda struggle in society [25] assumes that society is a struggle between two parties X and Y, each of which has its own media.

An individual belonging to this society, at each moment of time has a certain underlying position on the issue in question. This underlying position consists of a permanent attitude and the dynamic term. Attitude $\varphi \in (-\infty, \infty)$ is a fundamental tendency to support one party or another: it is formed during previous social experience of an individual and takes into consideration social situation and it is assumed to be unchanged for this confrontation. Dynamic component $\psi(t) \in (-\infty, \infty)$ has a meaning determined by social environment of the shift of stimuli towards the support of the party X. It is affected by the propaganda of both parties through the media and rumors, i.e., information shared by other members of the society through interpersonal communication.

The attitude φ is individual for each member of the society, as a dynamic component $\psi(t)$ describes the information field of society as a whole. (In this case, more complex models can take into account that, for example, conservatives read conservative newspapers more, and the liberals – the liberal newspapers. Model [16] takes into account that some companies do not use media, and receives information only in interpersonal communications).

If the internal position of some of the individual is positive $\varphi + \psi(t) > 0$, than his manifest position is to support party X, i.e. in interpersonal communication it spreads information to its advantage. Similarly, $\varphi + \psi(t) < 0$ corresponds to the support of the party Y. Thus, the underlying positions $\varphi + \psi(t)$ form a continuum, but manifest position is a binary value.

Let us denote through $N(\varphi)$ the function describing the distribution of individuals on the installation. In accordance with the above mentioned, the number $X(t)$ and $Y(t)$ of supporters of their respective parties, are given by

$$X(t)=\int_{-\psi(t)}^{\infty} N(\varphi)d\varphi,\ Y(t)=\int_{-\infty}^{-\psi(t)} N(\varphi)d\varphi. \qquad (2)$$

The total number of individuals is given by

$$\int_{-\infty}^{\infty} N(\varphi)d\varphi = N_0.$$

The model is based on Rashevsky neurological scheme [26,27], which describes the formation of the reaction of an individual in response to incoming stimuli taking into account his attitude. With regard to the subject of propaganda warfare between the two parties, the reaction is the manifest position of the individual, i.e. his participation in the spread of information in support of one of the parties. The stimulus is the information that he receives (both through interpersonal communication and mass media).

Very roughly, the formation reaction can be described as follows. Suppose someday the individual received information from three supporters of party X and a supporter of party Y, read one newspaper article in favor of party X and two in favor of the party Y. Having weighed these stimuli in a certain way, we get a change of his position in favor of a particular party (the weighting factors are set in this model exogenously). For example, taking into account his attitude, he could become more or less radical supporter of his own party or switch to the other side.

The model has the form (see [25]for details):

$$\frac{d\psi}{dt} = A\alpha\left[C\left(2\int_{-\psi(t)}^{\infty} N(\varphi)d\varphi - N_0\right) + b_1 - b_2\right] - a\psi, \qquad (3)$$

with the initial condition

$$X(0)=\int_{-\psi(0)}^{\infty} N(\varphi)d\varphi. \qquad (4)$$

Here $X(0)$ is given number of supporters of the party in the initial moment of time, the parameters $b_1 \ge 0$, $b_2 \ge 0$ characterize the intensity of the media propaganda of the parties, for clearness, in this work it is assumed that $b_1 > b_2$.

The positive constants α, a, A, C are introduced in the neurological model [26,27], which provides neurological sense for these parameters. We can also propose some sociological interpretation. Thus, the parameter <u>a</u> describes the decay rate of the dynamic term of the internal position of an individual. For example, in the hypothetical disappearance of all social stimuli, equation (3) gives $\psi(t) \sim \exp[-at]$, i.e. $\varphi + \psi(t) \sim \varphi + \exp[-at]$. Thus, the internal position of each individual is striving to the position without propaganda warfare. The parameter a is the rate of this "relaxation".

The constant C characterizes the importance of interpersonal communication compared to propaganda. For example, if individuals experience a strong distrust to media reports, but susceptible to the rumors, then $C >> 1$. Finally, the expression $A\alpha$ describes the general susceptibility of individuals to stimuli (in other words, characterizes the significance of the factors of confrontation in comparison with the attitude φ).

After finding the function $\psi(t)$ as the solutions of problem (2), it is easy to find the number of supporters $X(t), Y(t)$ for each of the parties. If for sufficiently large there is the inequality

$X(t) > Y(t)$ then we say that party X wins. If at the same time $\lim_{t\to\infty} X(t) = N_0$, the victory is called total. Similarly we introduce the concepts of victory and total victory of the party Y.

In the present work, the model (3),(4) is applied to the study of propaganda warfare in a polarized society. Thus, we study the situation in which the society consists of two groups, each of which has a tendency to support "their" source of information. More specifically, such distribution is considered (see Fig. 1)

$$N(\varphi) = \begin{cases} 0, & \varphi < -d-h \\ \dfrac{N_0}{4h}, & -d-h \le \varphi \le -d+h \\ 0, & -d+h < \varphi < d-h \\ \dfrac{N_0}{4h}, & d-h \le \varphi \le d+h \\ 0, & \varphi > d+h \end{cases} \qquad 5)$$

Here $d > h > 0$. The parameter d characterizes the degree of polarization of society that is how groups are distant from each other in attitudes. The parameter $1/h$ is a measure of consolidation of individuals within each group.

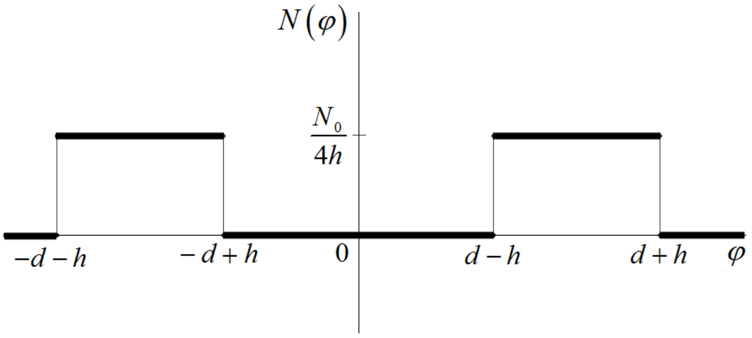


Fig. 1. The distribution of individuals $N(\varphi)$

Let's denote

$$f(\psi) = A\alpha\left[C\left(2\int_{-\psi(t)}^{\infty} N(\varphi)d\varphi - N_0\right) + b_1 - b_2\right].$$

To find the stationary solution to equation (3), let's consider all the possible ways of intersection of the graphs of functions $y = a\psi$ and $y = f(\psi)$.

We introduce the notation $P = A\alpha(b_1 - b_2)/a$, $Q = A\alpha CN_0/a$. It is easy to show that the mutual configuration of the line $a\psi$ and the broken $f(\psi)$ depends on the balance between the four numbers: Q, P, h and d.

In this work we restrict ourselves to a detailed analysis of the situation $h < Q/2 - P$, and a brief description of two other situations. In this case (see the remark after formula (5)) the polarized society is considered, i.e. $d > h$.

3. Analysis of stationary solutions

So, let the following inequality be satisfied

$$h < Q/2 - P. \tag{6}$$

The research strategy is to consider the various resulting cases in order of increasing polarization of society d, and on the basis of this analysis to make meaningful conclusions about the system.

Depending on the values of d, equation (3) has from 1 to 5 positions of equilibrium, for which we introduce the following notation:

$$\psi^1 = P + Q > 0,\ \psi^2 = \frac{Q(-d-h)+2hP}{2h-Q} > 0,\ \psi^3 = P > 0,$$

$$\psi^4 = \frac{Q(d-h)+2hP}{2h-Q} < 0,\ \psi^5 = -Q + P < 0.$$

The equilibrium position ψ^1 corresponds to a stationary number of supporters of political parties, equal to

$$\lim_{t\to\infty} X(t) = \int_{-\infty}^{\infty} N(\varphi)d\varphi = N_0,\ \lim_{t\to\infty} Y(t) = \int_{-\infty}^{-\psi^1} N(\varphi)d\varphi = 0.$$

This means a total victory of party X. Similarly for the other equilibria we have

ψ^2: $\frac{N_0}{2} < \lim_{t\to\infty} X(t) < N_0$, $0 < \lim_{t\to\infty} Y(t) < \frac{N_0}{2}$, incomplete victory of the party X

ψ^3: $\lim_{t\to\infty} X(t) = \lim_{t\to\infty} Y(t) = \frac{N_0}{2}$, draw

ψ^4: $\lim_{t\to\infty} X(t) < \frac{N_0}{2}$, $\frac{N_0}{2} < \lim_{t\to\infty} Y(t) < N_0$, incomplete victory of the party Y ψ^5: $\lim_{t\to\infty} X(t) = 0$, $\lim_{t\to\infty} Y(t) = N_0$, complete victory of the party Y

The semiaxis axle –это ось колеса в автомобиле, axis – это в математике, напр., координатная ось $d > h$ is cut by points $P+h$, $Q-P-h$, $Q+P-h$ on the segments, each of which has its own mutual configuration functions $y = f(\psi)$ and $y = a\psi$. Thus, we have the following cases.

1. If $h < d < P+h$, then the graph of functions $y = f(\psi)$ and $y = a\psi$ have three intersections (Fig. 2). Therefore, equation (3) has three stationary solutions, two of which (ψ^1 and ψ^5) are sustainable. The resulting equilibrium depends on the initial conditions: if $\psi(0) < \psi^4$ than the position of equilibrium ψ^5 is reached; if $\psi(0) > \psi^4$ than the equilibrium position - ψ^1.

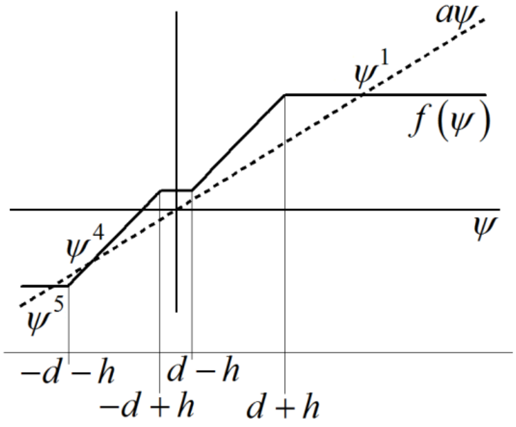


Fig. 2. Case 1: $h<d<P+h$

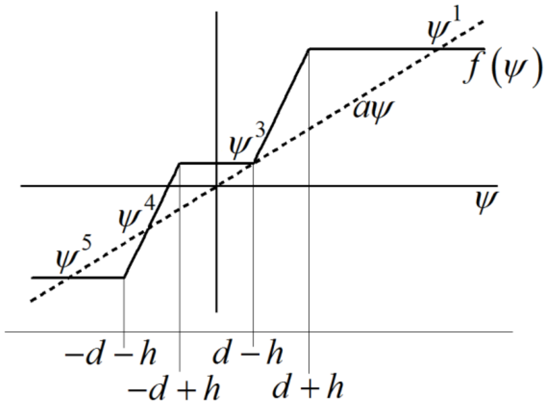


Fig. 3. Case 2: $d=P+h$

2. If $d=P+h$, then equation (3) have four stationary solutions (Fig. 3). If $\psi(0)>\psi^3$, then the position of equilibrium ψ^1 is achieved; if $\psi^4<\psi(0)\le\psi^3$, then ψ^3; if $\psi(0)<\psi^4$, then ψ^5.

3. If $P+h<d<Q-P-h$, then equation (3) has five stationary solutions (Fig. 4). If $\psi(0)>\psi^2$, then the position of equilibrium ψ^1 is achieved; if $\psi^4<\psi(0)<\psi^2$, then ψ^3; if $\psi(0)<\psi^4$, then ψ^5.

4. If $d=Q-P-h$, then equation (3) have four stationary solutions (Fig. 5). If $\psi(0)>\psi^2$, then the position of equilibrium ψ^1 is achieved; if $\psi^5<\psi(0)<\psi^2$, then ψ^3; if $\psi(0)\le\psi^5$, then ψ^5.

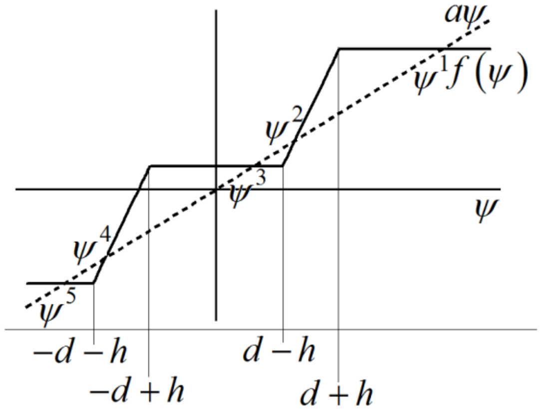


Fig. 4. Case 3: $P+h<d<Q-P-h$

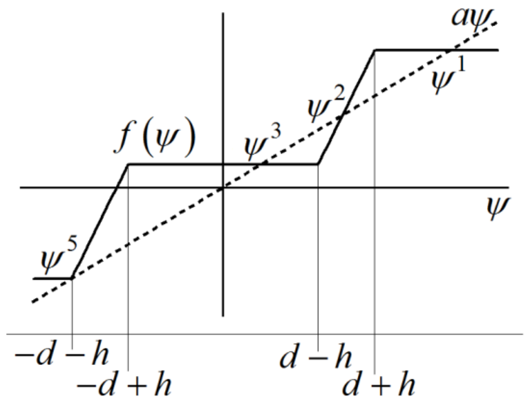


Fig. 5. Case 4: $d=Q-P-h$

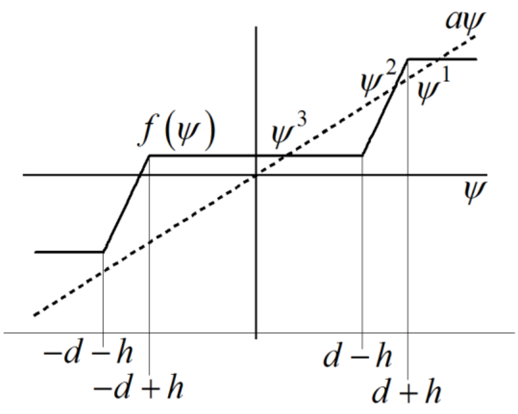


Fig. 6. Case 5: $Q-P-h<d<Q+P-h$

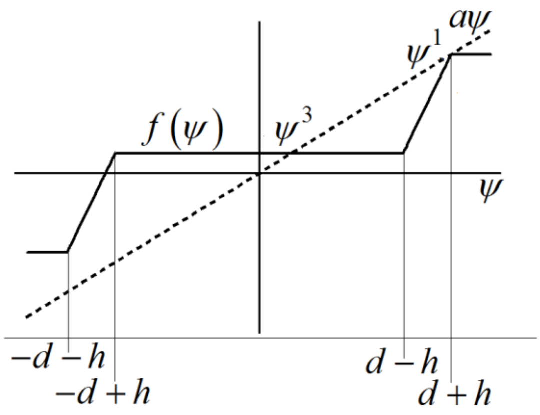


Fig. 7. Case 6: $d=Q+P-h$

5. If $Q-P-h<d<Q+P-h$, then equation (3) has three stationary solutions (Fig. 6). If $\psi(0)>\psi^2$, then the position of equilibrium ψ^1 is achieved; if $\psi(0)<\psi^2$, then ψ^3.

6. If $d=Q+P-h$, then equation (3) has two stationary solutions (Fig. 7). The resulting equilibrium depends on initial conditions: if $\psi(0)>\psi^1$ the position of equilibrium ψ^1 is reached; if $\psi(0)<\psi^1$, then ψ^3.

7. If $d>Q+P-h$, then equation (3) has one stationary solution ψ^3 (Fig. 8), which is achieved for any initial condition.

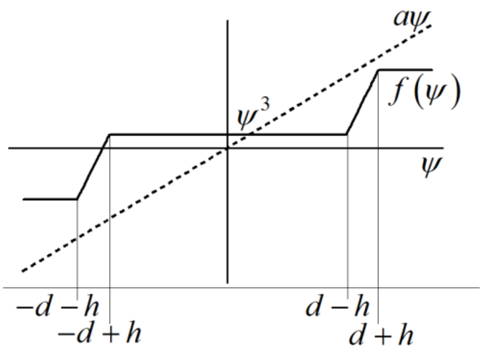


Fig. 8. Case 7: $d>Q+P-h$

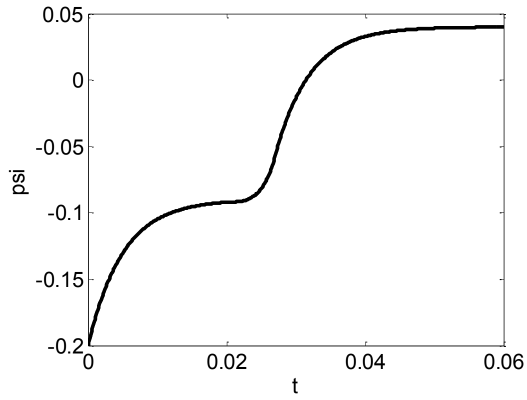
Thus, we've considered all the cases arising in the implementation of inequality (6). Sociological conclusions from this analysis are discussed in section 5.

4. The slow polarization of society process

In the previous section it was assumed that the polarization is constant. Now let's consider the case when it increases linearly over time. Such processes are connected with fundamental changes in the structure of society, so they are quite slow (compared to processes of information confrontation on specific issues). In this regard, let's assume the rate of increase in polarization as small:

$$d(t)=d_0+\varepsilon t, \tag{7}$$

where $\varepsilon << 1$. Numerical experiments show that the solutions of problem (3), (4) have an inner transition layer (Fig.18). Such decisions are called contrast structures (see, for example, [33-35]). The purpose of this section is the construction of asymptotics for solutions of this type. At the same time, we continue to assume that condition is fulfilled (6).



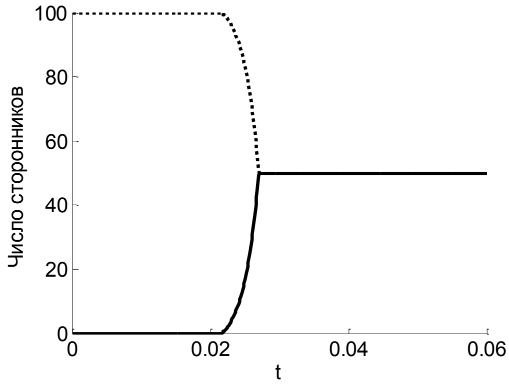


Fig. 9. On the left: the numerical solution to equation (3) with the following parameters: $a = 200$, $\alpha = 1$, $A = 4$, $N_0 = 100$, $C = 0.065$, $b_1 = 3$, $b_2 = 1$, $h = 0.02$, $d = 0.05$, $\varepsilon = 1$.

On the right: the number of supporters of party X (dotted line) and party Y (solid line) with the same parameters.

Having changed (3) variable $\theta = \varepsilon t$, we obtain

$$\varepsilon \frac{d\psi}{d\theta} = f(\psi, \theta) - a\psi, \tag{8}$$

$$X(0) = \int_{-\psi(0)}^{\infty} N(\varphi)\, d\varphi. \tag{9}$$

Here

$$f(\psi,\theta) = \begin{cases} A\alpha\left[-CN_0 + b_1 - b_2\right], & \psi \le -d_0 - \theta - h \\ A\alpha\left[\dfrac{CN_0}{2h}\left(\psi + (d_0 + \theta - h)\right) + b_1 - b_2\right], & -d_0 - \theta - h < \psi \le -d_0 - \theta + h \\ A\alpha\left(b_1 - b_2\right), & -d_0 - \theta + h < \psi \le d_0 + \theta - h \\ A\alpha\left[\dfrac{CN_0}{2h}\left(\psi + (h - d_0 - \theta)\right) + b_1 - b_2\right], & d_0 + \theta - h < \psi \le d_0 + \theta + h \\ A\alpha\left[CN_0 + b_1 - b_2\right], & \psi > d_0 + \theta + h \end{cases}. \tag{10}$$

Regarding the initial conditions let's assume for definiteness that there is the most meaningful case in which the society passes all of the states discussed in section 3, and the solution to the problem (8), (9) has the maximum possible number of transition layers. That is, let's assume

$$h < d_0 < P + h, \quad \psi(0) < -Q + P. \tag{11}$$

In accordance with the method of boundary functions [34] the asymptotic of solutions to problem (8)-(10) has the form

$$\psi(\theta,\varepsilon)=\psi_0(\theta)+\Pi_0\left(\frac{\theta}{\varepsilon}\right)+\Omega_0\left(\frac{\theta-\theta_0}{\varepsilon}\right)+\varepsilon\left[\psi_1(\theta)+\Pi_1\left(\frac{\theta}{\varepsilon}\right)+\Omega_1\left(\frac{\theta-\theta_0}{\varepsilon}\right)\right]+\dots \qquad (12)$$

Here θ_0 is the point of localization of the contrast structure (the corresponding value $t_0=\theta_0/\varepsilon$ is indicated in Fig.9), $\Pi_i(\theta/\varepsilon)$ and $\Omega_i((\theta-\theta_0)/\varepsilon)$ functions describing, respectively, the boundary layer in the neighborhood of the point $\theta=0$ and a transition layer in the neighborhood of the point $\theta=\theta_0$, $\psi_i(\theta)$ – regular members of the asymptotics ($i=0,1,2,\dots$).

Let's restrict ourselves to the construction of asymptotics of the zero order, and then compare the obtained result with the solution obtained by numerical methods.

The degenerate equation corresponding to (8), has the form

$$f(\psi,\theta)-a\psi=0. \qquad (13)$$

As follows from the analysis in section 3, for sufficiently small θ it has three roots: ψ^1, ψ^4, ψ^5, where ψ^1 and ψ^5 are stable and ψ^4 - unstable. If the conditions (11) are fulfilled the initial condition is in the area of influence of the root ψ^5. Thus $\psi_0(\theta)=\psi^5$, at $\theta<\theta_0$.

The value θ_0 is determined by the termination of the existence of the root ψ^5. This happens when $d(t)=Q-P-h$. Therefore, for θ_0 we have

$$\theta_0=Q-P-h-d_0. \qquad (14)$$

At $\theta>\theta_0$ and small enough difference $\theta-\theta_0$, degenerate equation (13) has three roots: ψ^1, ψ^2 and $\psi^3=P$, and ψ^1 and ψ^3 are stable, ψ^2 and – unstable. When $\theta\to\theta_0-0$ the solution of the problem (8)-(10) is in the neighborhood of the root ψ^5, i.e. it falls into the area of influence of the root ψ^3. In this case, since the root ψ^3 exists and is stable at all $\theta>\theta_0$, then $\psi_0(\theta)=\psi^3$ at all $\theta>\theta_0$.

So,

$$\psi_0(\theta)=\begin{cases}-Q+P, & 0\le\theta\le\theta_0\\ P, & \theta>\theta_0\end{cases}, \qquad (15)$$

where θ_0 is defined by expression (14). Thus, the regular part of asymptotics in the zero approximation is constructed.

Let's turn to the construction of the border function $\Pi_0(\theta/\varepsilon)$. Given the fact that $\theta/\varepsilon=t$, from (8)-(10) with (15) the equation for $\Pi_0(t)$:

$$d\Pi_0/dt=\Phi(\Pi_0)=$$

$$=\begin{cases}-a\Pi_0, & \Pi_0\le Q-P-d_0-h\\ a\left[\dfrac{Q}{2h}\left(-Q+P+\Pi_0+(d_0-h)\right)+Q-\Pi_0\right], & -d_0-h+Q-P<\Pi_0\le -d_0+h+Q-P\\ -a(-Q+\Pi_0), & -d_0+h+Q-P<\Pi_0\le d_0-h+Q-P\\ a\left[\dfrac{Q}{2h}\left(-Q+P+\Pi_0+(h-d_0)\right)+Q-\Pi_0\right], & d_0-h+Q-P<\Pi_0\le d_0+h+Q-P\\ a(2Q-\Pi_0), & \Pi_0>d_0+h+Q-P\end{cases} \qquad (16)$$

The initial condition for the function $\Pi_0(t)$ is determined from the condition

$$\psi_0(0)+\Pi_0(0)=\psi(0). \tag{17}$$

Hence we have

$$\Pi_0(0)=\psi(0)+Q-P. \tag{18}$$

From the condition (17) it follows that $\Pi_0(0)<0$. To define the function $\Phi(\Pi_0)$, let's note that

$$\Pi_0(0)-Q+P<-Q+P<-P-2h<-d_0-h$$

Therefore, to some right half-neighborhood of the point $\theta=0$ there is equality $\Phi(\Pi_0)=-a\Pi_0$, and equation (16) takes the form

$$\frac{d\Pi_0}{dt}=-a\Pi_0 \tag{19}$$

Solving it with initial condition (18), we obtain

$$\Pi_0(t)=\left(\psi^0+Q-P\right)e^{-at}. \tag{20}$$

This function takes values on half-neighborhood $\left[\psi^0+Q-P;0\right)$. Thus, for all values of $t>0$ we have $\Pi_0(t)-Q+P\le -d_0-h$, where, due to (16), we get that $\Phi(\Pi_0)=-a\Pi_0$. Thus, the equation (19) and the representation (20) are valid for all $t>0$.

Let us build the function of internal transition layer $\Omega_0\left((\theta-\theta_0)/\varepsilon\right)$. Let's introduce a variable $\tau=(\theta-\theta_0)/\varepsilon$. In addition, we will make an additional decomposition in a neighbourhood of a point θ_0: $\theta=\theta_0+\varepsilon\tau$. We have from (8)-(10) the equation for $\Omega_0(\tau)$:

$$d\Omega_0/d\tau=\Psi(\tau)=$$

$$=\begin{cases} a\left(-Q-\Omega_0\right), & \Omega_0\le -Q \\ a\left[\frac{Q}{2h}(\Omega_0+Q-2h+\varepsilon\tau)-\Omega_0\right], & -Q<\Omega_0\le -Q+2h \\ -a\Omega_0, & -Q+2h<\Omega_0\le Q-2P-2h \\ a\left[\frac{Q}{2h}\left(2P+2h+\Omega_0-Q\right)-\Omega_0\right], & Q-2P-2h<\Omega_0\le Q-2P \\ a\left(Q-\Omega_0\right), & \Omega_0>Q-2P \end{cases} \tag{21}$$

The initial condition for the function $\Omega_0(t)$ is determined from the condition

$$\psi_0(\theta_0)+\Omega_0(0)=\psi(\theta_0).$$

Hence we have

$$\Omega_0(0)=-Q \tag{22}$$

Therefore, to some right half-neighbourhood of the point $\theta=\theta_0$ the equation (21) takes the form

$$\frac{d\Omega_0}{d\tau}=a\left[\frac{Q}{2h}(\Omega_0+Q-2h+\varepsilon\tau)-\Omega_0\right].$$

Solving this equation with the initial condition (22), we find

$$\Omega_0(\tau)=\frac{2Q\varepsilon h}{a(Q-2h)^2}e^{\frac{a}{2h}(Q-2h)\tau}-\frac{Q\varepsilon}{Q-2h}\tau-\frac{2Q\varepsilon h}{a(Q-2h)^2}-Q.$$

The solution will have this form up to the moment τ_1, which is found from the condition

$$\frac{2Q\varepsilon h}{a(Q-2h)^2}e^{\frac{a}{2h}(Q-2h)\tau}-\frac{Q\varepsilon}{Q-2h}\tau-\frac{2Q\varepsilon h}{a(Q-2h)^2}=2h.$$

At $\tau>\tau_1$ equation (21) will be

$$\frac{d\Omega_0}{d\tau}=-a\Omega_0.$$

Solving this equation with initial condition

$$\Omega_0(\tau_1)=-Q+2h,$$

we get

$$\Omega_0(\tau)=(-Q+2h)e^{a\tau_1}e^{-a\tau}. \tag{23}$$

Since $(-Q+2h)e^{a\tau_1}e^{-a\tau}<(-Q+2h)e^{a\tau_1}<-2Pe^{a\tau_1}<0<Q-2P-2h$, then the representation (23) is valid for all $\tau>\tau_1$.

Thus, the zero approximation function $\psi(\theta)$ has the form

$$\psi(t)=\begin{cases} -Q+P+(\psi(0)+Q-P)e^{-at}+o(\varepsilon), & t\le\dfrac{\theta_0}{\varepsilon},\\ (\psi(0)+Q-P)e^{-at}+\dfrac{2Q\varepsilon h}{a(Q-2h)^2}e^{\frac{a}{2h}(Q-2h)(t-\theta_0/\varepsilon)}- & \\ -\dfrac{Q}{Q-2h}(\varepsilon t-\theta_0)-\dfrac{2Q\varepsilon h}{a(Q-2h)^2}-Q+P+o(\varepsilon), & \dfrac{\theta_0}{\varepsilon}<t\le\tau_1+\dfrac{\theta_0}{\varepsilon},\\ P+(\psi^0+Q-P)e^{-at}+ & \\ +(-Q+2h)e^{a\tau_1}e^{-a(t-\theta_0/\varepsilon)}+o(\varepsilon), & \theta>\varepsilon\tau_1+\dfrac{\theta_0}{\varepsilon}. \end{cases} \tag{24}$$

This asymptotics gives high accuracy of approximation to the solution even with small parameter ε values close to one. Fig. 10 shows the comparison of the zeroth approximation (24) and the solution obtained numerically.

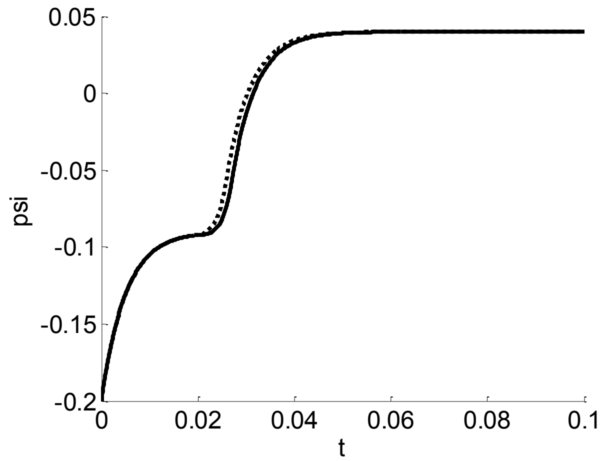


Fig. 10. Comparison of the solution obtained by numerical methods, and the zero-order approximation with the following parameters: $a = 200$, $\alpha = 1$, $A = 4$, $N_0 = 100$, $C = 0.065$, $b_1 = 3$, $b_2 = 1$, $h = 0.02$, $d = 0.05$, $\varepsilon = 1$, $\theta_0 = 0.2$, $\tau_1 = 0.0062$.

Here the solid line is the solution obtained by numerical methods, the dotted line - zero order approximation.

5. A sociological interpretation of the results

In this section, we consider a sociological interpretation to the mathematical results.

Fig. 11 serves to illustrate the case of inequality (6) which is considered in Section 3. For each value of polarization d and the initial condition $\psi(0)$ the outcome of the confrontation is specified: the complete victory of one party or another, or draw. If the polarization is small enough (i.e. $h < d < Q - P - h$), then the chances of parties to win are equal, despite the fact that the party X has propaganda of greater intensity, and a draw is also possible. Here and further "chances of winning" are understood in terms of the initial distribution of individuals between the parties, as shown in the figure. With the growth of polarization within these limits, the chances of each party to win are reduced and the chances for a draw increase. When the polarization exceeds the value $Q - P - h$ then party Y loses any chance to win, i.e. either the victory of party X, or draw is possible. At the same time, with further growth of the polarization, the chances of party X to win are reduced. Finally, at $d > Q + P - h$ only a draw is possible.

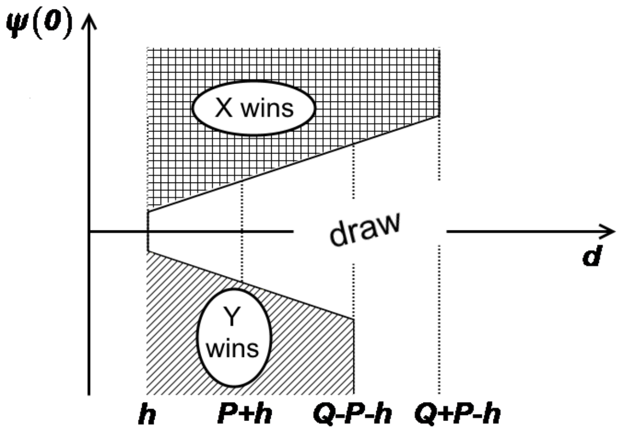
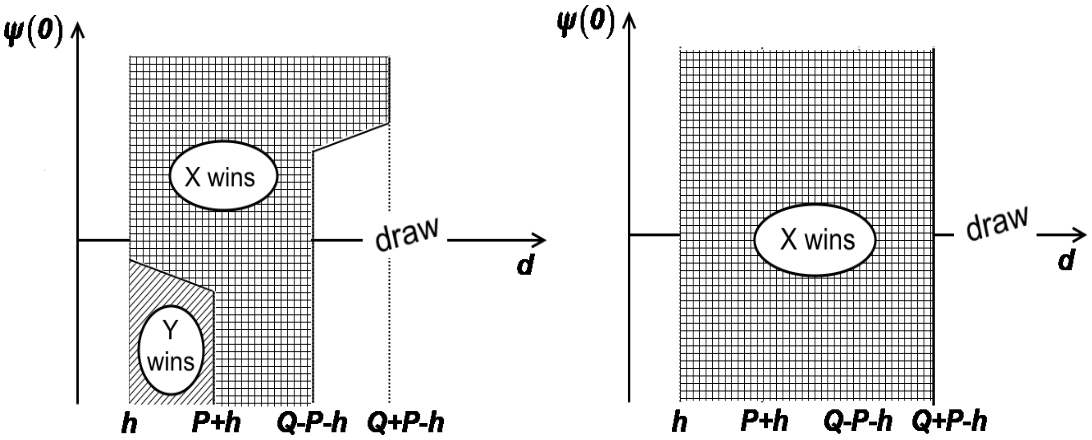


Fig. 11. The dependence of confrontation outcome on the initial conditions and the degree of polarization when the condition (6) is fulfilled

Condition (6) and connected with it Fig. 11 do not exhaust all possible cases with respect to the problem (3), (4). Graphs for the other two cases (out of six) are shown in Fig. 12. At the same time, the situation in Fig. 12 b is, in a sense, degenerative: the party with less intense propaganda cannot win under any initial conditions.

The graphs show that in all cases, at sufficiently high polarization of society the advantage in the propaganda becomes irrelevant, that is, the party with the stronger propaganda cannot win under any initial condition. Another conclusion – for this party, moderate polarization, as a rule is more favorable (and in any case no less favorable) than low polarization



a b

Fig. 12. The dependence of the information warfare outcome on the initial conditions and the degree of polarization; (a) $0.5Q - P < h < 0.5(Q - P)$, case (b) $0.5Q < h < 0.5(Q + P)$ $0.5Q < h < 0.5(Q + P)$

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Моделирование влияния политической поляризации на исход пропагандистского противоборства*

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Комплекс вопросов, связанных с пропагандистским противоборством и, более широко, с влиянием целенаправленно распространяемой информации на общество, привлекает неослабевающее внимание исследователей различных специальностей – социологов, специалистов в области информационных технологий и математиков. Модель выбора позиций индивидами при пропагандистском противоборстве применяется к изучению вопроса о влиянии степени поляризации общества на исход противостояния. Для этого рассмотрен случай распределения индивидов, соответствующего обществу, состоящему из двух групп, придерживающихся противоположных установок по некоторому вопросу. В настоящей работе развивается подход, акцентированный на модели информационного противоборства – именно, на выборе позиций индивидами при противоборстве. В основе этой модели лежит нейрологическая схема Рашевского.

Данная модель прилагается к изучению вопроса о том, как уровень политической поляризации влияет на исход пропагандистской борьбы. Данная постановка вопроса становится возрастающее актуальной, в первую очередь, в связи с развитием социальных медиа. В последнее время в литературе широко обсуждается вопрос об усилении поляризации в связи с развитием социальных медиа и вообще Интернета, а также о влиянии поляризации на политические события. В настоящей работе поляризованное общество описывается с помощью кривой распределения, имеющей два высоких горизонтальных плато. Расстояние между центрами тяжести этих плато принимается за меру поляризации. Таким образом, процесс возрастания поляризации имеет вид взаимного удаления плато друг от друга. Модель исследована аналитически и численно. Показано, что умеренная политическая поляризация благоприятствует стороне, имеющей превосходство в интенсивности пропаганды. Однако, если поляризация слишком сильна, то она нивелирует преимущество в пропаганде.

Ключевые слова: математическое моделирование, пропагандистское противоборство, нейрологическая схема Рашевского, конкурирующие информационные потоки.

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